How Rare Is Symmetry in Musical 12-Tone Rows?

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1. 12-TONE ROWS: MUSICAL AND GEOMETRIC SYMMETRY. Between 1914 and 1928, the Viennese composer Arnold Schoenberg developed a method for "12-tone" musical composition [**14**], [**13**], [**6**]. In a 12-tone composition, all harmonies and melodies are based on a 12-tone *row*. A row is an ordering of the set of twelve pitch chromas,[1] $\{C, C\sharp, D, D\sharp, E, F, F\sharp, G, G\sharp, A, A\sharp, B\}$, often represented by the set $\{0, 1, \ldots, 11\}$. The 12-tuple $(9, 10, 0, 3, 4, 6, 5, 7, 8, 11, 1, 2)$, for example, represents the 12-tone row for movement 5 of Schoenberg's Serenade, opus 24. Figure 1 displays this row in musical notation and as a *clock diagram* [**9**], [**1**]—a succession of arrows connecting the numbered vertices of a regular dodecagon.

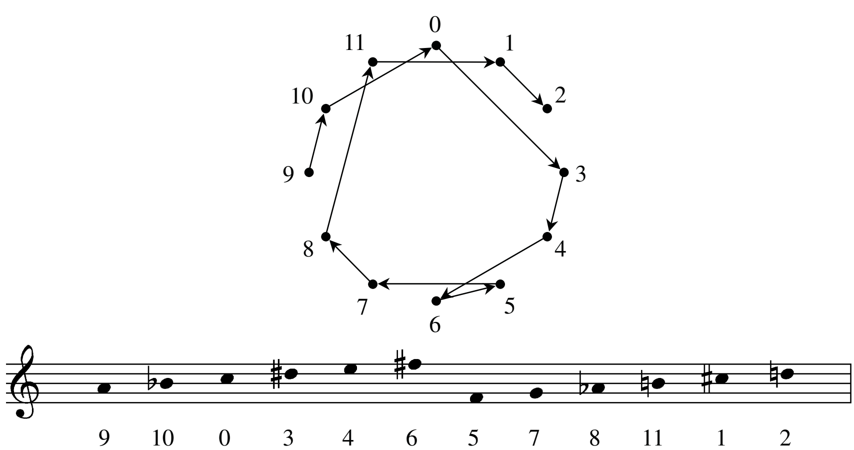

Figure 1. Clock diagram and musical notation for the row from Schoenberg's Serenade, opus 24, movement 5.

A 12-tone composition typically employs a row both in its *prime* form $p = (p_0, p_1, \ldots, p_{11})$ and in several transformations. Four methods for transforming a row will be discussed here: transposition, retrograde, inversion, and cyclic shift. Transposition, retrograde, and inversion are used throughout the Viennese 12-tone repertoire, whereas cyclic shift is confined to music by Schoenberg's student Alban Berg [**8**], and a few early pieces by Schoenberg himself [**7**]. Because cyclic shift is a rarer transformation, and because it complicates certain calculations, we prefer to treat it as a separate case.

Defining the transformations is straightforward. *Transposition* T_k rotates the diagram clockwise by $30k$ degrees (Figure 2, upper left). Musically, this corresponds to

[1] A *pitch chroma*, or *pitch class*, is the letter name of a pitch, without regard to the octave or height at which the pitch occurs. A low $C\sharp$ played on the cello and a high $C\sharp$ played on the piccolo share the same chroma, called $C\sharp$, $D\flat$, or 1. In both music and psychology, chromas are represented in a circular format [**15**], [**9**]; this justifies our use of clock diagrams.

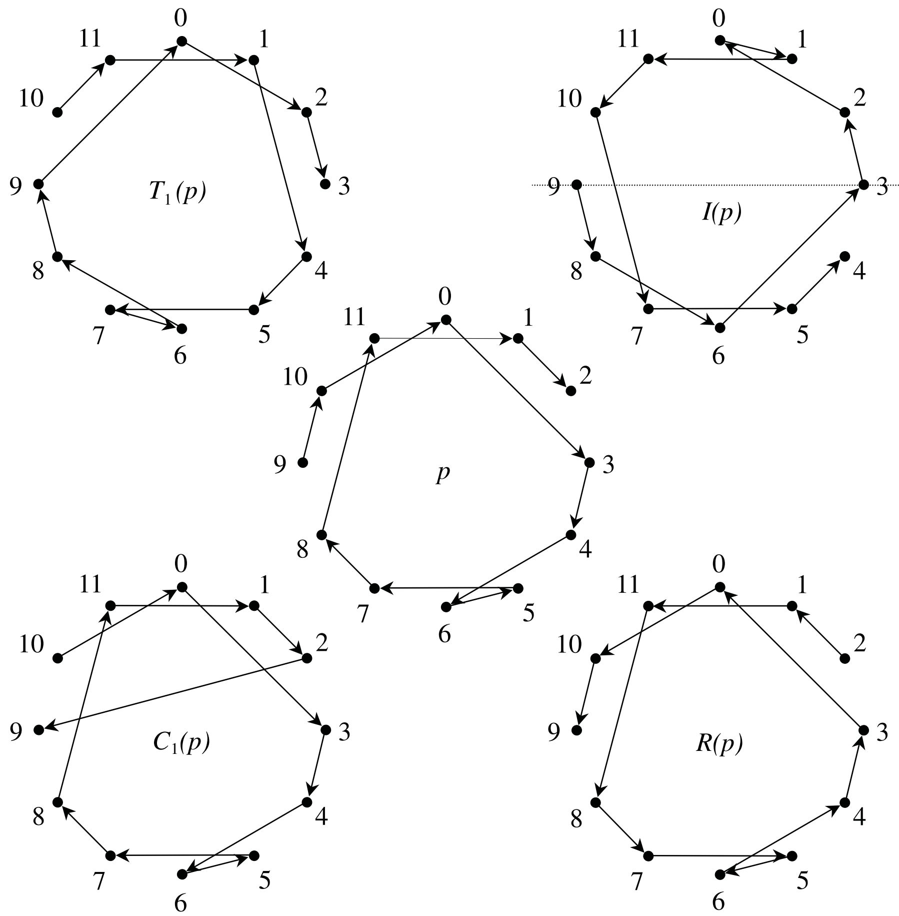

Figure 2. The row p from Figure 1 (center) transformed by transposition, inversion, retrograde, and cyclic shift (clockwise from upper left).

playing the row k semitones higher. On 12-tuples, T_k is given by the following map:

$$(p_0, p_1, \dots, p_{11}) \xmapsto{T_k} (p_0 + k, p_1 + k, \dots, p_{11} + k) \quad \text{mod } 12.$$

Inversion I reflects the clock diagram across the diameter containing the first note p_0 (Figure 2, upper right). Musically, this corresponds to playing the row upside-down. On 12-tuples, it becomes

$$(p_0, p_1, \dots, p_{11}) \xmapsto{I} (p_0, 2p_0 - p_1, \dots, 2p_0 - p_{11}) \quad \text{mod } 12.$$

Retrograde R reverses the arrows of the clock diagram (Figure 2, lower right), playing the row backwards:

$$(p_0, p_1, \dots, p_{11}) \xmapsto{R} (p_{11}, p_{10}, \dots, p_0).$$

Cyclic shift C_k moves the last k notes from the end of the row to the beginning:

$$(p_0, p_1, \dots, p_{11}) \xmapsto{C_k} (p_{0+k}, p_{1+k}, \dots, p_{11+k}),$$

in which the subscripts are taken modulo 12. On a clock diagram, cyclic shift deletes k arrows from the end of the sequence and adds k arrows to the beginning (Figure 2, lower left).[2]

Although Figure 2 depicts these transformations individually, they are also used in combination. Rows that are transformations of one another are called *equivalent* and are said to be part of the same *row class*. Under transposition, retrograde, and inversion, the row class may contain up to 48 different rows $(2 \cdot 2 \cdot 12)$. If cyclic shift is allowed as well, the size of the row class may grow to 576 $(48 \cdot 12)$.

Like rows, row classes can be represented geometrically. By removing the arrowheads and numbers from a clock diagram, we can represent a row class under transposition, retrograde, and inversion. The absence of arrowheads allows for equivalence under retrograde (reversal of arrows), and the absence of numbers allows for equivalence under transposition (rotation with respect to numbered vertices). To account for equivalence under inversion, we must also regard two diagrams as equivalent if one is a mirror image of the other. For the row in Figure 1, the row class appears in Figure 3.

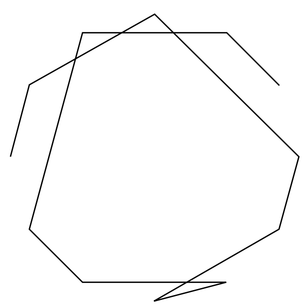

Figure 3. Row class for the row in Figure 1.

If we also allow cyclic shift, we can represent a row class by adding a final line segment to the diagram, forming a closed polygon with twelve vertices.[3] Because a polygon has no starting point, it cannot be altered by cyclic shift (change of starting point). Allowing for cyclic shift as well as the other three transformations, the row class for Berg's Violin Concerto is shown in Figure 4.

Composers sometimes choose a row that is *symmetric*. A symmetric row is one that is invariant under some transformation; geometrically, its row class has a symmetric diagram. The simplest type of symmetry occurs in the row for the Chamber Symphony by Schoenberg's student Anton Webern (Figure 5), which is its own transposed retrograde, i.e., $p = T_6(R(p))$. Such rows are called *palindromes* because, up to transposition, they are the same forward and backward. A palindrome's row-class diagram has rotational symmetry; the diagram for Webern's Chamber Symphony, for example, looks identical if rotated 180 degrees.

A more complicated type of symmetry occurs in the row from movement 5 of Schoenberg's Serenade, opus 24 (Figure 1); this row is its own transposed retrograde inversion, i.e., $p = T_5(R(I(p)))$. Rows like this, which are symmetric under three transformations (T, R, and I), have row-class diagrams with mirror symmetry. The

[2]We have borrowed the term cyclic shift from computer science. Music theorists often call it rotation, because it involves rotating the elements of the ordered list $(p_0, p_1, \ldots, p_{11})$. But this use of the word rotation can be confusing when clock diagrams are present. Rotating a clock diagram does *not* correspond to cyclic shift; instead, it corresponds to transposition.

[3]We allow nonsimple polygons, i.e., nonadjacent sides may intersect.

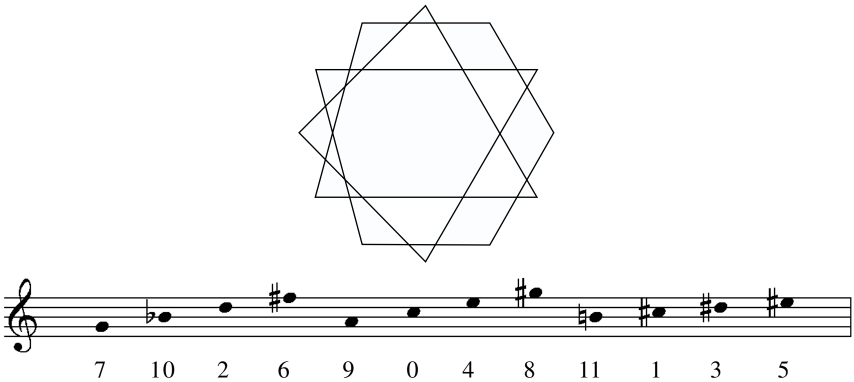

Figure 4. Polygon and musical notation for the row from Berg's Violin Concerto.

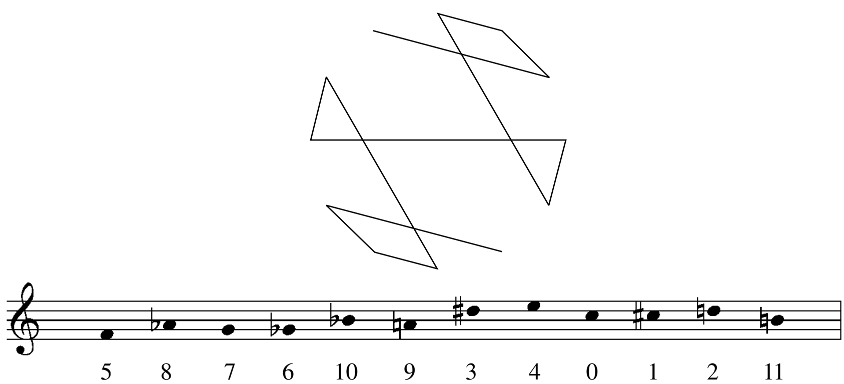

Figure 5. Palindromic row from Webern's Chamber Symphony, opus 21.

diagram for Schoenberg's Serenade, for example, looks like a brandy snifter, with mirror symmetry about the stem (Figure 3).

The most complicated type of symmetry occurs in the row from Berg's Violin Concerto (Figure 4). This row is symmetric under four transformations; namely, it is a transposed cyclic shift of its retrograde inversion, i.e., $p = T_4(C_3(R(I(p))))$. Again, the symmetry is evident from the row-class diagram, which has mirror symmetry around the horizontal. Note that this row is not symmetric without cyclic shift; in fact, removing any line from the diagram destroys the mirror symmetry.

Symmetric rows offer attractive musical possibilities. In the second movement of his Chamber Symphony, Webern combines a palindromic row (Figure 5) with palindromic rhythms and dynamics—creating a palindromic theme. In addition, symmetric rows can be broken into shorter segments that are similar in content. (Some nonsymmetric rows, called *derived rows*, share this property.) In Figure 1, for example, the musical notation shows how the row breaks into two six-note halves, each of which is a transposed retrograde inversion of the other. (Further inspection reveals that each six-note segment can be broken into two three-note halves, each of which is a transposed retrograde of the other. Thus, in addition to its ordinary symmetry, this row displays a kind of *nested* symmetry—a fine point that we will not dwell on here.)

It is widely believed that the Viennese 12-tone composers, especially Webern, had a penchant for symmetric rows [**13**], [**8**], [**3**]. Yet the number of symmetric rows used by these composers is not overwhelming. Under transposition, retrograde, and inversion, just four of the twenty-one row classes used by Webern, and two of the forty-two used by Schoenberg, are symmetric. In discussing the works of Berg, we should also include cyclic shift, since he used this transformation extensively [**10**]. But even so, only two of Berg's twenty-three rows qualify as symmetric.

Is the number of symmetric rows in Viennese 12-tone music large enough to evidence a taste for symmetry? As we intend to show, the answer is yes. Although symmetric rows are fairly rare in Viennese 12-tone music, they are rarer—much rarer—in the universe of all row classes.

We demonstrate the rareness of symmetric rows using the theory of permutation groups, a topic in elementary abstract algebra. The theory and calculations involved should be quite accessible to advanced undergraduates. In addition to traditional techniques, some calculations require the use of the symbolic algebra programming language `GAP` [**4**].

As well as demonstrating the rareness of symmetry, our methods provide a recipe for generating symmetric rows. This recipe may be useful to today's 12-tone composers.

2. AN ENUMERATION OF SYMMETRIC ROW CLASSES. While the geometric representation of rows helps clarify some issues involving symmetry, a precise enumeration of symmetric rows requires an algebraic representation. Algebraically, we can represent a 12-tone row as an element of S_{12}, the symmetric group on $\{0, 1, \dots, 11\}$. For example, the tone row from Schoenberg's Serenade is represented by the permutation $(0\ 9\ 11\ 2)(1\ 10)(5\ 6)$ in S_{12}. In this representation, we can model the four types of transformations as multiplication by certain elements of S_{12}.[4] Let $\tau = (0\ 1\ 2\ \cdots\ 11)$. For any tone row α in S_{12}, its transposition by one semitone is $\alpha\tau$ and its cyclic shift by one serial position is $\tau\alpha$.

Now let $\rho = (0\ 11)(1\ 10)(2\ 9)(3\ 8)(4\ 7)(5\ 6)$. For any α in S_{12}, $\rho\alpha$ gives the retrograde of α. Finally, let $\sigma = \rho\tau$. The inversion of α is $\alpha\tau^i\sigma\tau^{-i}$, where i is chosen so that $\alpha\tau^i$ fixes 0. In summary, the four transformations on α in S_{12} are as follows:

$$\alpha \overset{T_k}{\longmapsto} \alpha\tau^k, \quad \alpha \overset{I}{\longmapsto} \alpha\tau^i\sigma\tau^{-i}, \quad \alpha \overset{R}{\longmapsto} \rho\alpha, \quad \alpha \overset{C_k}{\longmapsto} \tau^k\alpha.$$

Let $D = \langle\tau, \rho\rangle$ (or, equivalently, $D = \langle\tau, \sigma\rangle$) and $F = \langle\rho\rangle$. Note that D is the dihedral group of order 24. For any tone row α in S_{12}, the set of all tone rows that can be obtained by applying a combination of retrograde, inversion, and transposition to α (i.e., the row class of α) is the double coset $F\alpha D$. If we also allow cyclic shift, the equivalence class becomes $D\alpha D$. We can detect a symmetric row by computing the size of these double cosets. For a row α with no symmetry, every nontrivial transformation yields a new row, so $|F\alpha D| = 48$ and $|D\alpha D| = 576$. However, if α exhibits symmetry under retrograde, inversion, and transposition, then $|F\alpha D| < 48$, and if a nontrivial composite of (some of) all four transformations fixes α, then $|D\alpha D| < 576$.

Denote the set of classes of tone rows equivalent under transposition, retrograde, and inversion (i.e., the set of row classes) by

$$\mathcal{T} = \{F\alpha D \mid \alpha \in S_{12}\}$$

and the set of equivalence classes under all four transformations (including cyclic shift) by

$$\mathcal{R} = \{D\alpha D \ \mid \ \alpha \in S_{12}\}.$$

[4] We multiply permutations left-to-right, so $\alpha\beta$ means do α first, then do β.

We now investigate the structure of $\mathcal{T}$ and $\mathcal{R}$. The number of row classes is provided by two propositions:

Proposition 1. $|\mathcal{T}| = 9{,}985{,}920$.

Proposition 2. $|\mathcal{R}| = 836{,}017$.

Proposition 2 was proved in [**5**] as the number of equivalence classes of paths traversing the vertices of a regular dodecagon. For an alternate derivation, see [**11**]. Proposition 1 appears in [**12**]. Whereas these sources give formulas for all n, we restrict our attention to the case $n = 12$. Generalization to all n is straightforward.

The proof of Proposition 1 in [**12**] is a nice application of Burnside's lemma, but our model provides a simpler derivation, using only standard facts from undergraduate algebra.

Proof of Proposition 1. For any α in S_{12}, $F\alpha D = \alpha D \cup \rho\alpha D$. Either $\alpha D \cap \rho\alpha D = \emptyset$ or $\alpha D = \rho\alpha D$; we shall count the number of α for which the latter occurs. Any such α has $\alpha^{-1}\rho\alpha$ in D. Since there are $12!/(2^6 \cdot 6!)$ conjugates of ρ, there are $2^6 \cdot 6!$ different αs that could conjugate ρ to any of its conjugates. Viewing D as the group of symmetries of a regular dodecagon, we see that there are seven conjugates of ρ in D: the six reflections over perpendicular bisectors of a side, along with rotation by 180°. Thus there are $2^6 \cdot 6! \cdot 7/24$ double cosets of size 24 in $\mathcal{T}$. The remaining cosets are all of size 48, so $|\mathcal{T}| = 2^6 \cdot 6! \cdot 7/24 + (12! - 2^6 \cdot 6! \cdot 7)/48$. ∎

This result provides insight into the rareness of symmetric tone rows. The $2^6 \cdot 6! \cdot 7$ αs for which $|F\alpha D| = 24$ are exactly the tone rows that can be transformed onto themselves by some composite of transposition, retrograde, and/or inversion. An argument similar to the foregoing proof reveals that $2^6 \cdot 6!$ of these tone rows exhibit symmetry under a composite of transposition and retrograde only. (These rows are palindromic, as defined earlier.)

Transposition and inversion are both right actions on S_{12}, and D is the group generated by these two actions, so the set of all rows that can be formed from a row α using transpositions and inversions is just the left coset αD. Since $|\alpha D| = |D|$ for all α, there are no tone rows symmetric via these two transformations alone. The corresponding geometric observation is that, although a mirror reflection may fix the lines of a clock diagram, it will reverse the arrows (see, for example, Figure 1).

Likewise, inversion and retrograde will not, by themselves, transform a tone row onto itself, because inversion fixes the first note and retrograde does not. Thus the remaining $2^6 \cdot 6! \cdot 6$ symmetric tone rows are mapped onto themselves by a composite of transposition, retrograde, and inversion. (The row in Figure 1 is an example.)

Table 1 enumerates the symmetric row classes under transposition, retrograde, and inversion.

TABLE 1. Rareness of symmetry under transposition, retrograde, and inversion.

Symmetry	Coset size	# of cosets	% of cosets	# of rows	% of rows
T, R	24	1,920	0.019%	46,080	0.0096%
T, R, I	24	11,520	0.115%	276,480	0.058%
none	48	9,972,480	99.87%	478,679,040	99.93%

Under transposition, inversion, and retrograde, symmetric tone rows are quite rare. If we also allow cyclic shift, the situation gets harder to analyze; however, we can calculate the sizes of the double cosets $D\alpha D$ using the symbolic algebra programming language GAP [**4**]. Table 2 lists the different coset sizes that occur, along with their frequencies. Table 2 indicates that symmetry under all four transformations (indicated by a coset size less than 576) is also quite unusual, although not as rare as it is without cyclic shift.

TABLE 2. Rareness of symmetry under transposition, inversion, retrograde, and cyclic shift.

Coset size	# of cosets	% of cosets	# of rows	% of rows
24	2	0.00024%	48	0.00001%
48	2	0.00024%	96	0.00002%
72	6	0.00072%	432	0.00009%
96	17	0.0020%	1632	0.00034%
144	152	0.018%	21,888	0.0046%
192	11	0.0013%	2,112	0.00044%
288	8,545	1.02%	2,460,960	0.514%
576	827,282	98.84%	476,514,432	99.48%

Table 2 also shows that very small coset sizes are especially rare. For example, only two cosets have size 24; these represent highly symmetric structures that are important in music theory. One is the *chromatic scale*, e.g., (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11), whose diagram is the dodecagon. The other is the *circle of fifths*, e.g., (0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5), whose diagram is the dodecagram. (In music a "perfect fifth" is seven semitones, so the circle of fifths is defined with $p_i = 7 + p_{i-1} \bmod 12$.)

The GAP code for generating Table 2 is straightforward. To calculate the double cosets $D\alpha D$ for α in S_{12}, execute the following:

```
G:=SymmetricGroup(12);
D:=Group((1,2,3,4,5,6,7,8,9,10,11,12),(1,12)(2,11)(3,10)
          (4,9)(5,8)(6,7));
x:=DoubleCosetRepsAndSizes(G,D,D);;
```

(Note that GAP uses the symbols 1, . . . 12 instead of 0, . . . , 11.) The list x is a collection of pairs: the first item of each pair is the coset representative, and the second item is the size of the coset. The following functions return the total number of cosets, the number of cosets of size 192, and a list of the coset representatives for all the size 192 cosets.

```
Size(x);
c192:=Filtered(x, L -> L[2]=192);;
Size(c192);
List(c192, L -> L[1]);
```

Given a representative of a double coset, the elements of the double coset are easy to compute. For example,

```
AsList(DoubleCoset(D,(2,6)(3,11)(4,8)(5,9),D));
```

lists all of the elements in the coset $D\alpha D$ for $\alpha = (1\ 5)(2\ 10)(3\ 7)(4\ 8)$. Thus it is straightforward to implement our mathematical recipe for generating symmetric rows.

3. SYMMETRIC ROW CLASSES IN VIENNESE 12-TONE MUSIC. Under transposition, retrograde, and inversion, symmetric row classes constitute just 0.13% of the universe of possibilities. Yet they constitute 5% of the row classes in Schoenberg (2 of 42) and 20% of the row classes used by Webern (4 of 21). It seems clear that these composers liked symmetry. If they had chosen row classes at random, without regard to symmetry, it is improbable that either composer would have used such a large number of symmetric row classes. (For Schoenberg, the probability would be .0015; for Webern, 1.25×10^{-8}.)[5]

Even when cyclic shift is allowed, just 1.16% of all row classes are symmetric. Yet 9% of the rows used by Berg display this property (2 of 23). Again, the probability is low (.024) that Berg would have used so many symmetric row classes by chance alone.[6]

The *types* of symmetry used by the Viennese composers are also of interest. Neither of the symmetric rows used by Schoenberg, and only one of the four symmetric rows used by Webern, is a palindrome (symmetric under transposition and retrograde alone). The rareness of palindromes in Webern has led one scholar to infer that Webern preferred non-palindromic symmetry [**3**]. We cannot endorse this view, however, since even in the universe of symmetric rows palindromes are outnumbered 6 to 1 (Table 1). One of Berg's symmetric rows, by the way, is also a palindrome; the other, as remarked earlier, is symmetric under all four transformations (see Figure 4).

Although Viennese tone rows display above-chance levels of symmetry, more than 90% of them are not symmetric. Evidently most rows were chosen on the basis of other criteria. This conclusion agrees with the judgment of music scholars, who have suggested several other row types that may have interested these composers—including "combinatorial," "all-interval," and "tonally colored" rows [**2**], [**13**].

Because Viennese composers had a variety of preferences, it can be hard to know whether a row is symmetric because the composer wanted symmetry, or because the composer wanted another property that happens to be related to symmetry. For example, both Webern (once) and Schoenberg (twice) used rows that are symmetric only when cyclic shift is allowed [**10**]. At first, this seems surprising: Webern never used cyclic shift, and Schoenberg had abandoned it by the time of his cyclically symmetric rows. However, both Schoenberg and Webern favored "hexachordally derived" rows that could be broken into similar six-note halves [**2**]; here, that preference led to rows that are symmetric using 6-fold cyclic shift (C_6).

Because other preferences may be confounded with symmetry, it seems reasonable to ask whether Viennese rows are remarkably symmetric *given the constraints imposed by the composers' other preferences*. This question is hard to answer algebraically; however, an approximate answer can be obtained using statistical methods. First, one generates rows from (say) 1000 randomly selected row classes.[7] Then, using a statistical classification technique, one attempts to discriminate these random row classes from those actually used by Viennese composers. Results using this approach suggest that, even when other preferences are accounted for, Viennese tone rows still display a significant penchant for symmetry [**16**].

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[5]These probabilities are based on a hypergeometric distribution with $n = 9{,}985{,}920$ row classes of which $r = 13{,}440$ are symmetric (as in Table 1). For example, if Schoenberg had drawn 42 row classes from this distribution, the probability that 2 or more would be symmetric is .0015.

[6]This probability is based on a hypergeometric distribution with $n = 836{,}017$ row classes of which $r = 8{,}735$ are symmetric (as in Table 2).

[7]The simplest approach is to generate random rows, i.e., random permutations of $\{0, 1, \ldots, 11\}$. However, this will tend to underweight the symmetric rows, which have smaller row classes (cosets). One must compensate for this underweighting, or avoid it by sampling row classes directly.

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